On closeness between an entire function of completely regular growth and its Phragmen-Lindelof indicator

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Let $\rho(r)$ be a proximate order, $\rho(r) \to \rho$, $r \to \infty$. Let f be an entire function of proximate order ρ, that is $\log \max\{|f(z)| : |z| = r\} = O(r^{\rho(r)})$, $r \to \infty$. The function

$$h_f(\theta) = \limsup_{r\to\infty} \frac{\log |f(re^{i\theta})|}{r^{\rho(r)}}$$

is called the *indicator of* f. The indicator is a ρ-trigonometrically convex function (being a constant for $\rho = 0$).

An entire function f is called an *entire function of completely regular growth* ([1]) if

$$\log |f(re^{i\theta})| = h_f(\theta) r^{\rho(r)} + \varepsilon(re^{i\theta}) r^{\rho(r)},$$

where $\varepsilon(re^{i\theta})$ tends to 0 uniformly outside E as $re^{i\theta} \to \infty$, and E is a C_0-*set*, i.e. $E \subset \bigcup_k D(z_k, r_k)$, $\quad \sum_{|z_k|\le r} r_k = o(r), \quad r \to \infty$.

We are interested in the interplay between estimates outside exceptional sets of $\varepsilon(re^{i\theta})$ and the zero distribution of f.

In the case when all zeros are located on a finite number of rays emanating from the origin and $\varepsilon(re^{i\theta}) = O(|z|^{\rho_1-\rho})$ as $|z| \to \infty$ outside some exceptional set E of values z, for some $\rho_1 < \rho$, the problem was solved in [2] and [3]. We consider the general case using an approach from [4].

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