Widom factors for generalized Jacobi measures

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Let K be a regular compact subset of $[-1,1]$ and μ_K be its equilibrium measure. For a given weight w on K, let $T_{n,w}^{(K)}$ denote the n-th weighted Chebyshev polynomial with respect to w on K and $P_n(\cdot;\mu)$ denote the n-th monic orthogonal polynomial for a finite Borel measure μ with $\text{supp}(\mu)=K$. Define

$$W_{\infty,n}(K,w) := \frac{\|w\, T_{n,w}^{(K)}\|_K}{\text{Cap}(K)^n} \tag{1}$$

and

$$W_{2,n}(\mu) := \frac{\|P_n(\cdot;\mu)\|_{L_2(\mu)}}{\text{Cap}(K)^n}. \tag{2}$$

We discuss optimal upper and lower bounds for $W_{\infty,n}(K,w)$ if $w(x) = \sqrt{1-x^2}, \sqrt{1-x}$ or $\sqrt{1+x}$. We also investigate optimal lower bounds for $W_{2,n}(\mu)$ for the cases $d\mu(x) = (1-x^2)d\mu_K(x)$, $d\mu(x) = (1-x)d\mu_K(x)$ and $d\mu(x) = (1+x)d\mu_K(x)$.